control case of Ref. 21. These results further demonstrate the basic realizability of the triplet grouping of the MRAS adaptation rules in Refs. 19-21. The existence of this triplet sequence is further shown by the work of Shahein,[22] Ten Cate,[23] and Colburn and Boland.[24]

The additional terms in Eq. (8) can allow for increased flexibility in error convergence rate control. By proper selection of the f, g, ρ, and σ terms, an improved convergence rate could result. This could be of great importance if one wishes to use the identifier results for further control operations or where stability information is crucial, as in power systems. It has been shown[25,26] that for complete analysis, interpretation of a criterion surface in $2n$ dimension parameter space is needed. However, such work is beyond the scope of this Note. Also, although Eqs. (13-16) require differentiation, which is undesirable from a noise viewpoint, variations in filtering and problem structure modification could be employed for purposes of implementation.

V. Conclusions

Two new adaptive identification laws were developed and asymptotic stability proved using Lyapunov theory. Using an approximate analysis employing a linearized error characteristic equation approach, it was shown how the two methods are analogous to a previously published method as regards classical P-I-D type control laws. Due to differentiation, if noise is present there may be practical problems with regard to tracking accuracy and stability. An advantage of the two methods presented is the possible improvement in the transient tracking response, as shown by a linearized error response analysis.

Some interesting areas for future work include: 1) determining an exact transient stability analysis approach, as in Refs. 17 and 25, and 2) determining the effect of relaxing the "frequency richness" input requirement on the size of the parameter tracking errors $\|\phi\|$, $\|\psi\|$ (since asymptotic stability of the parameter estimates is no longer assured).[27]

Acknowledgment

This work was supported in part by AFOSR Grant No. 77-3295.

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Stability Augmentation by Eigenvalues Control and Model Matching

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Introduction

THE high-performance requirements of modern aircraft have led to the development of digital fly-by-wire control systems based on model-following techniques.[1-4] In such techniques, a model embodying all desired performance

Received Oct. 6, 1977; revision received March 22, 1978.

Index category: Handling Qualities, Stability and Control.

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requirements is constructed. The aircraft control actuators are then operated in such a way that the aircraft dynamics follow the specified model. Generally, when performing certain maneuvers, the aircraft stability is considerably degraded, thus limiting its performance capabilities. Utilization of stability augmentation system permits the aircraft to perform the required maneuver with the desired stability requirements.

The proposed stability augmentation system could be considered as a version of a digital fly-by-wire control system. The present scheme differs fundamentally from the conventional model-following techniques. It requires knowledge of the model eigenvalues rather than the model coefficient matrix. A new, simple yet powerful approach for eigenvalues assignment is presented. It is valid for distinct as well as repeated eigenvalues, complex or real. The control vector is linearly related to the system state and to the command input through two gain matrices. The state-related gain matrix is computed so that the system closed loop eigenvalues match those of the model.

Formulation

In continuous form, the actual state is given by

$$\dot{x} = Fx + Gu \tag{1}$$

where x is an n-dimensional state vector and u is an m-dimensional control vector. F and G are properly dimensioned coefficient matrices. The model state, in continuous form, is given by

$$\dot{x}_m = F_m x_m + G_m \delta_c \tag{2}$$

where m refers to the model.

The model state has the same dimension as the actual state, and the command input vector δ_c has the same dimension as the control vector u. The control vector may be adequately represented by the relation

$$u = Mx + N\delta_c \tag{3}$$

where M and N are state and command input gain matrices which, in general, may be time varying. Substituting Eq. (3) into Eq. (2) yields

$$\dot{x} = (F + GM)x + GN\delta_c \tag{4}$$

A comparison of Eqs. (2) and (4) shows that the actual system will match the model if the following relations are satisfied:

$$F + GM = F_m \tag{5a}$$

$$GN = G_m \tag{5b}$$

The first of the preceding equations matches the transient response of the system to that of the model, while the second equation matches the steady-state conditions. The command input gain matrix could be readily obtained by satisfying Eq. (5b). The main objective of the present work is to produce a software technique for system stability augmentation. This could be achieved by properly calculating the state-related gain matrix M, such that the system closed loop eigenvalues match the preassigned model ones. In this sense, the stability qualities rather than the performance characteristics of both the system and the model become identical. This is because the eigenvectors of the closed-loop system coefficient matrix may be different from those of the model. Our interest, therefore, will be focused on computing the incremental product matrix GM by the eigenvalues control method presented in the following section.

Eigenvalues Control

The eigenvalues of the open-loop coefficient matrix F are the roots of the characteristic equation

$$\det(F - \lambda I) = 0 \tag{6}$$

Denoting the coefficients of the resulting nth degree polynomial by $c_{n-1}, c_{n-2}, \ldots, c_0$, it is clearly verified that those coefficients are expressed by combinations of linear products of matrix F elements, f_{ij}; $i,j = 1,\ldots,n$. In functional form, this may be written as

$$c_p = c_p(f_{ij}) \qquad p = n-1, n-2, \ldots, 0 \qquad i,j = 1,\ldots,n \tag{7}$$

Equation (6) may be written in vector form as

$$c^T v = 0 \tag{8}$$

where vectors c and v are given by:

$$c^T = [1 \; c_{n-1} \; c_{n-2} \ldots c_0]$$

$$v^T = [\lambda^n \; \lambda^{n-1} \ldots \lambda \; 1]$$

Define the roots (eigenvalues) of Eq. (8) by e_k; $k = 1,\ldots,n$. In general, the roots are considered complex, i.e., $e_k = e_{kR} + ie_{kI}$, where R and I denote the real and imaginary parts, respectively.

In terms of the system open-loop eigenvalues, the characteristic equation may be written in the alternate form:

$$\prod_{k=1}^{n} (\lambda - e_k) = 0 \tag{9}$$

Changes in the eigenvalues will be produced by changes in the elements of the coefficient matrix. Denoting the eigenvalue changes by Δe_k; $k = 1,\ldots,n$, and the corresponding changes in the elements of the coefficient matrix by Δf_{ij}; $i,j = 1,\ldots,n$, we get from Eqs. (8) and (9):

$$\prod_{k=1}^{n} [\lambda - (e_k + \Delta e_k)] = (c + \Delta c)^T v \tag{10}$$

where

$$\Delta c^T = \Big[0 \sum_{i=1}^{n} \sum_{j=1}^{n} (\partial c_{n-1}/\partial f_{ij}) \Delta f_{ij} \sum_{i=1}^{n} \sum_{j=1}^{n} (\partial c_{n-2}/\partial f_{ij}) \Delta f_{ij} \ldots \sum_{i=1}^{n} \sum_{j=1}^{n} (\partial c_0/\partial f_{ij}) \Delta f_{ij} \Big] \tag{11}$$

Equating coefficients of like powers of λ on both sides of Eq. (10), will result in n equations. Such n equations, when decomposed into real and imaginary parts, become $2n$ equations. The eigenvalues may be complex or real, while the coefficients c_k; $k = n-1, n-2, \ldots, 0$, as well as the elements f_{ij}; $i, j = 1,\ldots,n$, are real. The resulting equations may be represented by the functional forms:

$$g(e_{kR}, e_{kI}, \Delta e_{kR}, \Delta e_{kI}) = c + \Delta c \qquad k = 1,\ldots,n \tag{12a}$$

$$h(e_{kR}, e_{kI}, \Delta e_{kR}, \Delta e_{kI}) = 0 \tag{12b}$$

where g and h are n-dimensional vector functions.

For a specific dynamic system, e_{kR} and e_{kI} are the real and imaginary parts of the eigenvalues of the open-loop coefficient matrix F, which could be readily calculated. Since the complex eigenvalues are always in conjugate pairs, and any desired shift in the real or imaginary parts should be done for the conjugate pairs simultaneously, it could be verified that for any system, regardless of its order, Eq. (12b) is always satisfied identically.

Equation (12a) represents a system of n equations with n^2 unknowns, namely, Δf_{ij}; $i,j=1,...,n$. For Eq. (12a) to be mathematically determinate, at least n^2-n of the incremented elements Δf_{ij} must be set to zero. This leaves, at the most, n of the incremented elements to be computed, leading to the following important conclusion. The eigenvalues of a given $(n\times n)$ matrix could be shifted by perturbing, at the most, n of its elements. We then denote the nonzero incremented elements of matrix F by Δf_q^*; $q=1,...,r\leq n$. Substitution of Eq. (11) into Eq. (12a) leads to:

$$\Delta f^* = W^{-1}[-c+g(e_{kR},e_{kI},\Delta e_{kR},\Delta e_{kI})] \tag{13}$$

where W is an n-square matrix given by

$$W=\begin{bmatrix} \partial c_{n-1}/\partial f_1^* & \partial c_{n-1}/\partial f_2^* & \cdots & \partial c_{n-1}/\partial f_n^* \\ \partial c_{n-2}/\partial f_1^* & \partial c_{n-2}/\partial f_2^* & \cdots & \partial c_{n-2}/\partial f_n^* \\ \hline \partial c_0/\partial f_1^* & \partial c_0/\partial f_2^* & \cdots & \partial c_0/\partial f_n^* \end{bmatrix} \tag{14}$$

and

$$\Delta f^{*T}=[\Delta f_1^* \quad \Delta f_2^* \quad \cdots \quad \Delta f_n^*]$$

Equation (13) shows that the vector Δf^* has a unique value if matrix W is nonsingular. In other words, the eigenvalues of the system could be controlled whenever there is a vector Δf^*, such that matrix W is nonsingular. Such a condition will guide to the proper selection of the elements of vector Δf^*.

Stability Augmentation

The system could be controlled to yield the desired closed-loop eigenvalues by different choices of elements Δf_q^*; $q=1,...,r\leq n$, of matrix GM [see Eq. (5a)]. Choice of such elements is arbitrary, if the sensitivity matrix W is nonsingular. However, at most, n out of the n^2 elements of the product matrix GM are nonzeros; therefore elements of vector Δf^* are to be selected in accordance with physically realizable gain matrix M.

Applications

Two examples will be considered in this section. The first one is a simple second-order system, where the presented concept of eigenvalue control is illustrated. The second example is that of augmenting the stability of an aircraft during a pull-up longitudinal maneuver.

Example 1

Consider a second-order system whose coefficient matrix has the elements $f_{11}=-1$, $f_{12}=1$, $f_{21}=-2.25$, $f_{22}=0$. The eigenvalues of the given systems are $-0.5\pm i\,1.4142$.

For a general second-order system, Eq. (12a) becomes

$$-\Delta f_{11}-\Delta f_{22}=f_{11}+f_{22}-(e_{1R}+e_{2R}+\Delta e_{1R}+\Delta e_{2R}) \tag{15}$$

$$f_{22}\Delta f_{11}-f_{21}\Delta f_{12}-f_{12}\Delta f_{21}+f_{11}\Delta f_{22}=f_{12}f_{21}-f_{11}f_{22}$$
$$+(e_{1R}+\Delta e_{1R})(e_{2R}+\Delta e_{2R})-(e_{11}+\Delta e_{11})(e_{21}+\Delta e_{21}) \tag{16}$$

For the present example, three cases are considered: case 1, shifting the real part to -0.6; case 2, shifting the imaginary parts to ± 1.2142, and case 3, shifting both the real and imaginary parts to $-0.6\pm i\,1.2142$. The calculated eigenvalues resulting from different chosen incremented elements for the three cases are shown in Table 1.

Table 1 Calculated eigenvalues for different choices of incremental elements

Incremented elements	Increments		Calculated eigenvalues
	Case 1		
f_{12}, f_{22}	$\Delta f_{12}=-0.04002$,	$\Delta f_{22}=-0.2$	$-0.6\pm i\,1.4142$
f_{21}, f_{22}	$\Delta f_{21}=0.09004$,	$\Delta f_{22}=-0.2$	$-0.6\pm i\,1.4142$
f_{11}, f_{22}	$\Delta f_{11}=-0.09004$,	$\Delta f_{22}=-0.1099$	$-0.6\pm i\,1.4177$
	Case 2		
f_{11}, f_{12}	$\Delta f_{11}=0$,	$\Delta f_{12}=-0.2336$	$-0.5\pm i\,1.2142$
f_{11}, f_{21}	$\Delta f_{11}=0$,	$\Delta f_{21}=0.5257$	$-0.5\pm i\,1.2142$
f_{11}, f_{22}	$\Delta f_{11}=-0.5257$,	$\Delta f_{22}=0.5257$	$-0.5\pm i\,1.0945$
	Case 3		
f_{12}, f_{22}	$\Delta f_{12}=-0.2736$,	$\Delta f_{22}=-0.2$	$-0.6\pm i\,1.2142$
f_{21}, f_{22}	$\Delta f_{21}=0.6157$,	$\Delta f_{22}=-0.2$	$-0.6\pm i\,1.2142$
f_{11}, f_{21}	$\Delta f_{11}=-0.2$,	$\Delta f_{21}=0.4157$	$-0.6\pm i\,1.2142$
f_{11}, f_{12}	$\Delta f_{11}=-0.2$,	$\Delta f_{12}=-0.1847$	$-0.6\pm i\,1.2142$
f_{11}, f_{22}	$\Delta f_{11}=-0.6157$,	$\Delta f_{22}=0.4157$	$-0.6\pm i\,1.1038$

Example 2

Aircraft dynamics during a pull-up maneuver may be approximated by the equation:

$$\dot{x}=\begin{bmatrix} -1 & 1 & 0 \\ -2.25 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 22.5 \\ 0 \end{bmatrix} u$$

The open loop eigenvalues are $e_1=0$, $e_{2,3}=-0.5\pm i\,1.41421$. It is necessary to shift the real part of the second and third eigenvalues to -0.6, i.e., the desired model eigenvalues are 0, $-0.6\pm i\,1.41421$. The model control coefficient matrix is:

$$G_m^T=[0 \quad -20 \quad 0]$$

Because the Δf_{ij} are the elements of the matrix (F_m-F), choice of the nonzero incremental elements should be consistent with Eq. (5a). For the given problem, the nonzero incremental elements are Δf_{21}, Δf_{22}, and Δf_{23}. Using the preceding data Eq. (12a) for third-order systems gives $\Delta f_{21}=0.09$, $\Delta f_{22}=-0.2$, $\Delta f_{23}=0$.

Thus the gain matrices M and N are

$$M=[0.004 \quad .00889 \quad 0] \qquad N=-0.88889$$

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